Progression in Written Addition

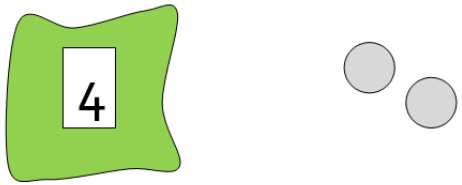
Stage 1 - Concrete

Children are taught that addition is the combining of two or more amounts. They begin by counting all of the items in the groups.

To find out how many altogether, touch and drag them into a line one at a time whilst counting.

Stage 2 - Concrete

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted.

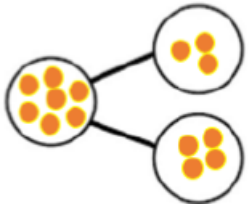


For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Stage 3 – Pictorial

Children are able to use a part, part, whole model to record the addition calculation.

They are able to draw representations of the numbers and use the methods in stage 1 and stage 2 to find the total.



Stage 4 – Abstract

Children are able to write a number sentence to record the calculation they have solved using the methods in stage 1, 2 and 3.

For example – 4 + 2 = 6 and 4 + 3 = 7

It is important that children understand the concept of addition before moving on from this stage.
They should know and understand that it is:

- **Combining two or more groups to give a total or sum**
- **Increasing an amount**
- **Commutative**

Stage 5 - Concrete

Children move on to using Base 10 equipment to support their developing understanding of addition.

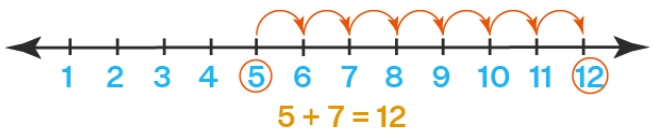
11 + 5 = 16

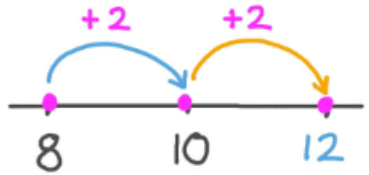
11 cubes are lined up (1 ten and 1 unit/one).
5 cubes are added to the line of 11 giving a total of 16.

If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

Stage 6 – Pictorial

Children use a number line to count on to find the total. They may count on in steps of 1 or more. Children should be encouraged to add on to make 10 and then add the remaining.



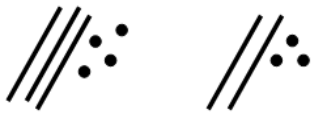


Progression in Written Addition

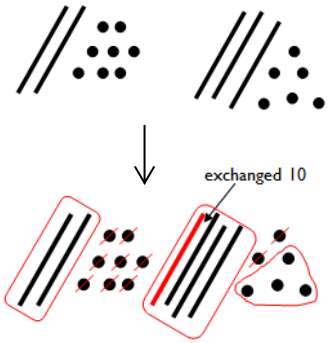
Stage 7 – Concrete/Pictorial

Children continue to use the Base 10 equipment to support their calculations, including exchanging 10 units/ones for 1 ten when the total of the units/ones is 10 or more. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

34 + 23 = ?
The units/ones are added first 4 + 3 = 7
The tens are added next 30 + 20 = 50
Both answers are put together 50 + 7 = 57

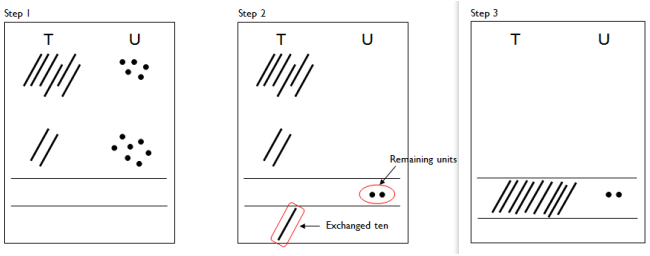


28 + 36 = ?
The units/ones are added first
8 + 6 = 14 with ten units/ones exchanged for 1 ten.
A ring is put around the units/ones not exchanged – this is the units part of the answer.
The tens are then added, including the exchanged ten, to complete the sum



Stage 8 – Concrete/Abstract

65 + 27

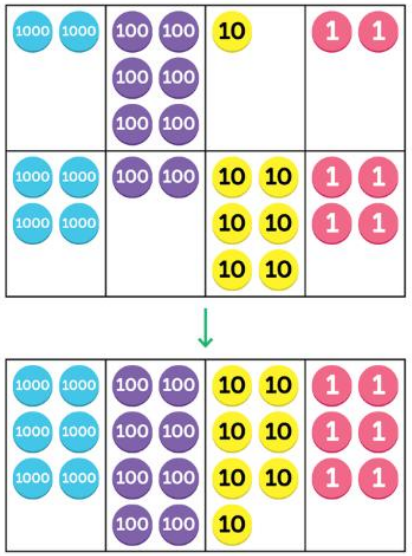


Written method

Step 1	Step 2	Step 3
T U	T U	T U
6 5	6 5	6 5
+ 2 7	+ 2 7	+ 2 7
	2	9 2
	1	1

Stage 9 – Concrete/Pictorial/Abstract

Children use place value counters to support their calculations, including exchanging 10 ones for 1 ten when the total of the ones is 10 or more. They will record their own drawings of the place value counters using circles.



	2	6	1	2
+	4	2	6	4
	6	8	7	6

Stage 10 - Abstract

HTU			
625	367	321	£3.48
+ 48	+ 85	+ 7	+ £0.78
673	452	+ 48	£4.26
1	11	376	1 1
		1	

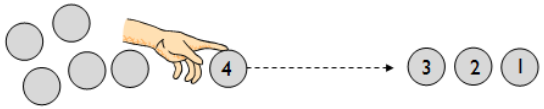
This is the final stage of the method, and should be continued to be used for all written addition calculations.
The example top left would be 'said' as follows:
5 + 8 = 13, put 3 down and carry the 10
20 + 40 + 10 that was carried over = 70 *(7 written in the tens column)*
600 + 0 = 600 *(6 written in the hundreds column)*

Children will be expected to use this method for adding numbers with more than 3 digits, numbers involving decimals and adding any number of amounts together.

Progression in Written Subtraction

Stage 1 - Concrete

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. 9 – 4.



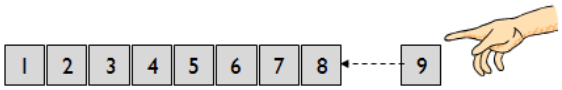
Stage 2 - Concrete

Children move on to using Base 10 equipment alongside a number track to support their developing understanding of subtraction. **13 - 4 = ?**

13 cubes are lined up.
4 cubes are removed from the end of the line leaving 9 left.
It is important that children keep track of how many have been removed.

Touch count and remove the number to be taken away.

Touch count to find the number that remains.



Stage 3 – Pictorial

Children are able to draw the concrete resources e.g. circles for counters or squares for base 10 equipment. They can then cross out the number they are taking away and count how many are left.

6 – 3 = ?

Stage 4 – Abstract

Children are able to write a number sentence to record the calculation they have solved using the methods in stage 1, 2 and 3.

For example – 10 – 6 = 4 and 15 – 7 = 8

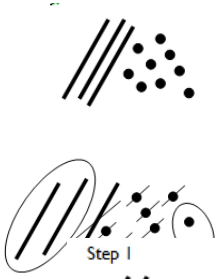
It is important that children understand the concept of subtraction before moving on from this stage.
They should know and understand that it is:

- **A removal of an amount from a larger group (take away).**
- **Comparison of two amounts (difference).**

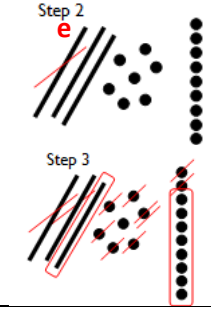
Stage 5 – Concrete/Pictorial

Children continue to use the Base 10 equipment to support their calculations. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

39 - 17 = ?
39 is drawn
17 is crossed out
A ring is drawn around what is left
to give the answer giving 22

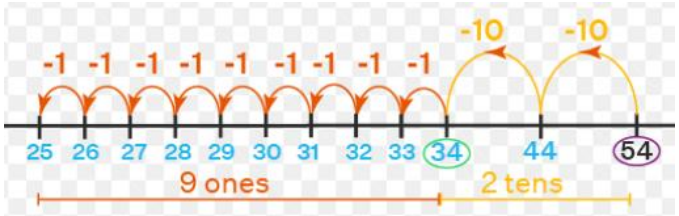


37 – 19 = ?
37 is drawn
9 units cannot be crossed out, so
a ten is crossed out and exchanged
for 10 ones which are in a line.
e is written next to the exchanged ten.
19 is crossed out
A ring is drawn around what is left
to give the answer 18



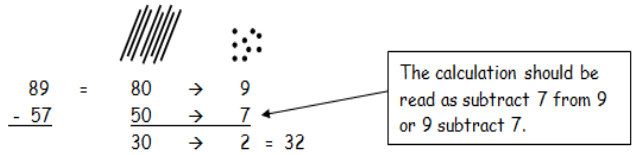
Stage 6 – Pictorial

Children use a number line to count back to take away. They may count back in steps of 1 or more.



Progression in Written Subtraction

Stage 7 – Concrete/Pictorial/Abstract



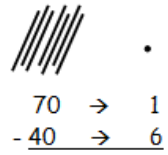
Children move from using the Base 10 method to expanded vertical method, using base 10 notation and arrow cards. Children learn to subtract the least significant digits first (start with the numbers on the right and work from right to left).

The answer to each individual subtraction is written underneath before these answers are recombined.

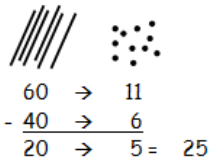
Stage 8 – Concrete/Pictorial/Abstract

This stage involves exchange.

It is clear that there are not enough units to subtract 6 from 1, so one of the tens from the 70 is exchanged for 10 ones.



The initial number 71 is rearranged as 60 and 11 to make the calculation easier.



This would be recorded by the children as:

60
~~7~~0 → 11
- 40 → 6
20 → 5 = 25

Stage 9 – Abstract

This final stage is the compact method of decomposition.

754
- 86

6141
~~7~~~~5~~4
- 86
668

This is the final stage of the process and will continue to be used with larger numbers and numbers involving decimals

Progression in Written Multiplication

Stage 1 – Concrete/Pictorial

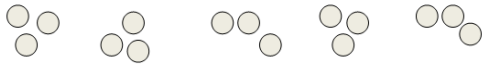
Children are encouraged to develop a mental image of the size of numbers. They learn to think about equal groups or sets of objects in practical, real life situations.
They begin to record these situations using pictures.

A child's jotting showing fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

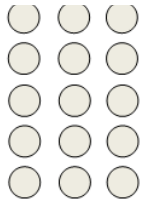
Stage 2 – Concrete/Pictorial

Children understand that multiplication is repeated addition and that can be done by counting in equal steps/groups.



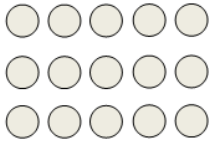
or

Children can then be introduced to the image of a rectangular array, initially through real items such as egg boxes, baking trays, ice cube trays, wrapping paper etc. and using these to show that counting up in equal groups can be a quicker way of finding a total.



3 + 3 + 3 + 3 + 3 = 15

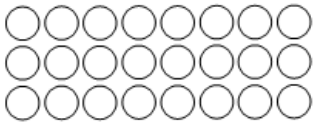
Children also understand that 3 x 5 is the same as 5 x 3



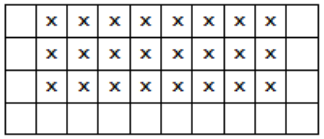
5 + 5 + 5 = 15

Stage 3 – Concrete/Pictorial

Children continue to use arrays and create their own to represent multiplication calculations



3 x 8 = 8 + 8 + 8 = 24



3 x 8 = 8 + 8 + 8 = 24

Stage 4 – Concrete/Pictorial/Abstract

The children use base 10 equipment to support their calculations. They will use their own drawings of Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

23 x 3 =

20 x 3 =

3 x 3 =

Stage 5 - Abstract

The expanded method. The place value columns are labelled to ensure children understand the value of each digit in the original number and the answer.

Th	H	T	U	
	3	6	8	
x			6	
	4	8		(8 x 6)
	3	6	0	(60 x 6)
+ 1	8	0	0	(300 x 6)
	2	2	0	8

Stage 6 – Abstract

The formal method. The place value columns are still labelled.

Th	H	T	U
	3	6	8
x			6
2	2	0	8
	4	4	

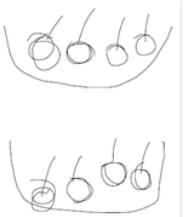
Progression in Written Division

Stage 1 – Concrete/Pictorial

Children are encouraged to develop a mental image of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving **equal** groups and **equal** sharing.

They may develop ways of recording calculations using pictures.

A child's jotting showing halving six spots between two sides of a ladybird.



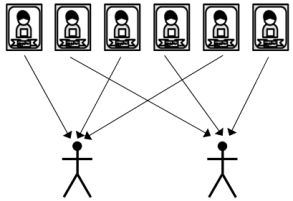
A child's jotting showing how they shared the apples at snack time between two groups.

Stage 2 – Concrete/Pictorial

Children explore practical contexts where they share equally and group equally. 6 ÷ 2 = ?

Equal sharing (6 shared equally between 2)

6 football stickers are shared equally between 2 people, how many do they each get? Children may solve this by using a 'one for you, one for me' strategy until all of the stickers have been given out.



Equal grouping (How many groups of 2 are there in 6?)

There are 6 football stickers, how many people can have 2 stickers each?

Stage 3 – Concrete/Abstract

Children continue to use practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation.

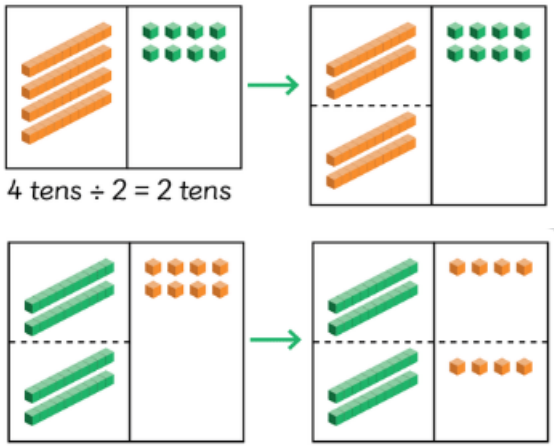
12 ÷ 3 = ? Children begin to read this calculation as, 'How many groups of 3 are there in 12?'

At this stage, children will also be introduced to division calculations that result in remainders.

13 ÷ 4 = 3 remainder 1

Stage 4 – Concrete/Abstract

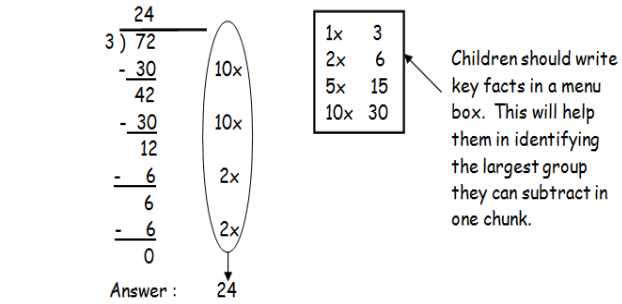
The children will use partitioning to partition a number before they divide. E.g. 48 ÷ 2 =



Stage 5 - Abstract

This is the 'chunking' method of division in which children use key facts of the multiplication tables of the divisor.

72 ÷ 3



Stage 6 - Abstract

During this stage children should become more efficient when using the chunking method by not having any subtraction steps that repeat a previous step.

196 ÷ 6